BAREISS DETERMINANT ALGORITHM, pen and paper approach

1) Start with a square matrix M:

```
   2  3  1  1
   2  1  3  3
   2  3  1  3
   1  2  2  1
```

2) Add a floating “1” to the upper left. We will call this $M_{0,0}$ to fit a pattern that will emerge.

```
1
   2  3  1  1
   2  1  3  3
   2  3  1  3
   1  2  2  1
```

3) Create a Swaps Counter, which starts at 0. We need to keep track of how many times we swap rows as go through the algorithm.

```
1
   2  3  1  1  Swaps: 0
   2  1  3  3
   2  3  1  3
   1  2  2  1
```

4) Circle row 1 and column 1 (represented here by highlighting).

```
1
   2  3  1  1  Swaps: 0
   2  1  3  3
   2  3  1  3
   1  2  2  1
```

5) Add a clone of this matrix to the right, except all values of $M_{i,j}$ to the bottom right of the circled row / column will initially be blanks.

```
1
   2  3  1  1  Swaps: 0    2  3  1  1
   2  1  3  3              2  _  _  _
   2  3  1  3              2  _  _  _
   1  2  2  1              1  _  _  _
```

6) Fill in each of those blanks by running this calculation on the first matrix and putting the result in $M_{i,j}$ of the second matrix:

$$\frac{M_{i,j} * M_{1,1} - M_{1,j} * M_{i,1}}{M_{0,0}}$$

1

```
2   3   1   1   Swaps: 0    2   3   1   1
2   1   3   3               2  -4   4   4
2   3   1   3               2   0   0   4
1   2   2   1               1   1   3   1
```

(The point of circling the row and column is perhaps clearer now, so it's easier to keep track of which matrix elements to multiply by.)

7) We can ignore that first matrix now and concentrate on the second one. We will also retain our Swaps counter, which is still at 0:

```
2   3   1   1   Swaps: 0
2  -4   4   4
2   0   0   4
1   1   3   1
```

8) Circle the second row and second column:

```
2   3   1   1   Swaps: 0
2  -4   4   4
2   0   0   4
1   1   3   1
```

9) Clone the matrix as before, blanking out the lower right entries:

```
2   3   1   1   Swaps: 0    2   3   1   1
2  -4   4   4               2  -4   4   4
2   0   0   4               2   0   _   _
1   1   3   1               1   1   _   _
```

10) Fill in each of those blanks by running this calculation on the first matrix and putting the result in $M_{i,j}$ of the second matrix:

$$\frac{M_{i,j} * M_{2,2} - M_{2,j} * M_{i,2}}{M_{1,1}}$$

```
2   3   1   1   Swaps: 0    2   3   1   1
2  -4   4   4               2  -4   4   4
2   0   0   4               2   0   0  -8
1   1   3   1               1   1  -8  -4
```

11) Now we focus again on our new matrix, and again we keep our Swaps counter:

```
2  3  1  1  Swaps: 0
2 -4  4  4
2  0  0 -8
1  1 -8 -4
```

12) Circle the third row and column:

```
2  3  1  1  Swaps: 0
2 -4  4  4
2  0  0 -8
1  1 -8 -4
```

13) We have run into a snag: $M_{3,3}$ is 0, and that's unacceptable for a diagonal value. So what we will do is swap row 3 with one of its successors (there is only one in this case) that has a non-zero value in column 3. We will also increment our Swaps counter:

```
2  3  1  1  Swaps: 1
2 -4  4  4
1  1 -8 -4
2  0  0 -8
```

14) Clone and blank as before:

```
2  3  1  1  Swaps: 1   2  3  1  1
2 -4  4  4             2 -4  4  4
1  1 -8 -4             1  1 -8 -4
2  0  0 -8             2  0  0  _
```

15) Fill the remaining cells (just one cell) per this formula:

$$\frac{M_{i,j}*M_{3,3} - M_{3,j}*M_{i,3}}{M_{2,2}}$$

```
2  3  1  1  Swaps: 1   2  3  1  1
2 -4  4  4             2 -4  4  4
1  1 -8 -4             1  1 -8 -4
2  0  0 -8             2  0  0 -16
```

16) FINAL RESULT: The determinant is the value in that lower right cell, times -1 for every swap we performed.

-16 * -1 = **16**